October 8, 2008 WOCOMAL Varsity Meet

Round 1: Arithmetic (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $x=3$, $y=2$, and $z=5$, evaluate $\left(\dfrac{xyz}{x-y+z}\right)^4$.

2. For every two roses I buy at the regular price, I get a third rose for \$1.00. I spent \$23.20 for a dozen roses. Find the regular price of one rose.

3. a and b are two real numbers. If $a \otimes b = ab+7a+7b+42$, find a number e such that $a \otimes e = a$ for all values of a.

ANSWERS

(1 pt.) 1. ______________________

(2 pts.) 2. ______________________

(3 pts.) 3. ______________________

Burncoat, Millbury, South

October 8, 2008 WOCOMAL Varsity Meet

Round 2: Algebra I (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. A sporting goods store sells T-shirts for one price and sweatshirts for a different price. John spent the same amount on 5 T-shirts as he spent on 2 sweatshirts. He spent a total of \$115. What is the cost of each T-shirt?

2. Simplify: $(x+1)(x+2)(x+3)-(x+2)(x+3)(x+4)+3(x+2)(x+3)$

3. The wind force (w) on a vertical surface varies directly as the product of the area (a) of the surface and the square of the wind velocity (v). When the wind blows at 30 mph the force on a 10 sq ft area is 45 lbs. Find the force on this area when the wind blows at 80 mph.

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. ____________________ lbs

Auburn, Doherty, Shepherd Hill

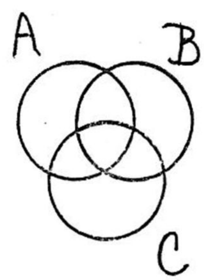
October 8, 2008 WOCOMAL Varsity Meet

Round 3: Set Theory (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $A \cup B = \{2,7\}$, $A \cap B = \{2\}$, and $B \cup C = \{1,2,3,4,5,6\}$, find A.

2. Set A has 12 more subsets then set B. How many elements are there in set A?

3. 

These 3 Venn circles divide their union into 7 non-overlapping regions. The number of elements in the 7 regions are 7 consecutive counting numbers. Using the same 7 consecutive counting numbers, find the differences between the maximum number of possible elements in circle A and the minimum number of possible elements in circle A.

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

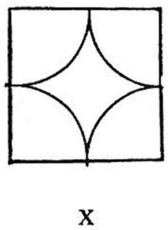
(3 pts.) 3. ____________________

Shrewsbury, Tantasqua, Worcester Academy

October 8, 2008 WOCOMAL Varsity Meet

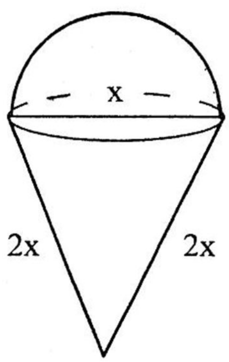
Round 4: Measurement (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. 

A star is made by cutting quarter circles from a square sheet of cardboard of side x. What is the area of the star in terms of x? Leave answer in terms of π.

2. What is the area of a circle that has a 12 inch chord 4 inches from the center of the circle? Leave answer in terms of π.

3. 

A hemisphere (with diameter x) of ice cream sits on a cone (with diameter x and slant height of 2x) as shown. If all of the ice cream melts into the cone, what fraction of volume of the cone does the ice cream occupy? (Assume density of melted and frozen ice cream is the same.)

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________ in^2

(3 pts.) 3. ____________________

Auburn, Bromfield, St. John's

October 8, 2008 WOCOMAL Varsity Meet

Round 5: Polynomial Equations (No Calculators)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find all values of x for which $\left(x^2-6x+4\right)^2=16$.

2. If m and n are non-zero solutions of $x^2+m^2x+n^2=0$, find the numerical value of m.

3. Given $x^3-2x^2+3x-4=0$. Find the sum of the reciprocals of the roots of the equations.

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. ____________________

Algonquin, Burncoat, Leicester

October 8, 2008 TEAM ROUND WOCOMAL Varsity Meet

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND
ON THE SEPARATE TEAM ANSWER SHEET (2 points each)

1. Evaluate: $\dfrac{3(2)-4\left[2^3(3)-(-1)^6\right]-2^2}{-2^4+2^0}$

2. Solve for a: $(3a-4)^2-12=(4a+3)(a-2)+14$

3. Universal set $U=\{a,b,c,d,e,f,g\}$, $B=\{a,c,e,g\}$, and $C=\{b,e,f,g\}$
Find $B' \cup C$ where B' is the complement of B

4. A square of side S and a rectangle whose length is L have the same area. The perimeter of the rectangle equals the circumference of a circle whose radius is R. Express π in terms of these 3 variables: S, L, and R.

5. The polynomial $f(x)$ has exactly three zeroes at $x=3$, $x=-\dfrac{3}{4}$, and $x=\dfrac{5}{2}$. If $f(2)=66$, find $f(-4)$.

6. If each year of the 21st century were written in Roman numerals, which year would require the most Roman numerals? Give your answer in modern 4-digit form.

7. Find the last seven digits of the counting number 101^9.

8. Find the sum of all positive integers n for which $\left(4x^n+\dfrac{x^{-3}}{2}\right)^8$ will have a term that is x-free.

9. A rectangle is broken down into nine squares with bases measuring 1, 4, 7, 8, 9, 10, 14, 15, and 18 units respectively. What were the dimensions of the original rectangle?

Assabet Valley, Auburn, Doherty, Hudson, St John's, Westborough, West Boylston, Worcester Academy

October 8, 2008

WOCOMAL Varsity Meet ANSWERS

Round 1: Arithmetic

(1 pt.) 625

(2 pts.) $2.40

(3 pts.) -6

Round 2: Algebra I

(1 pt.) $11.50

(2 pts.) 0

(3 pts.) 320 lbs

Round 3: Set Theory

(1 pt.) $\{2,7\}$ (need both values with $\{\ \}$)

(2 pts.) 4

(3 pts.) 12

Round 4: Measurement
(leave answers in terms of π)

(1 pt.) $x^2-\pi\left(\frac{x}{2}\right)^2$ or $x^2\left(1-\frac{\pi}{4}\right)$ or other equivalent forms

(2 pts.) $52\pi\ in^2$

(3 pts.) $\frac{2\sqrt{15}}{15}$

Round 5: Polynomial Equations

(1 pt.) 0, 2, 4, 6 (need all four answers, any order)

(2 pts.) -2

(3 pts.) $\frac{3}{4}$ or 0.75

October 8, 2008 TEAM ROUND ANSWERS WOCOMAL Varsity Meet

(2 points each)

1. 6

2. 4, $-\frac{1}{5}$ or -0.2 (need both 4 and $-\frac{1}{5}$ or equivalent)

3. $\{b,d,e,f,g\}$ (need all in any order, including parenthesis)

4. $\pi = \frac{L^2+S^2}{RL}$ or $\frac{L^2+S^2}{RL}$

5. -7098

6. 2088

7. 4,360,901

8. 39

9. 32 by 33 units or 33 by 32 units

---------------- Round 1: Arithemetic ---------------

1. $\left(\dfrac{(2)(3)(5)}{3-2+5}\right)^4=\left(\dfrac{(6)(5)}{6}\right)^4=5^4=\boxed{625}$.

3. $a\otimes e=ae+7a+7e+42=a$

$e(a+7)=-6a-42$

$e=\dfrac{-6(a+7)}{(a+7)}=\boxed{-6}$

2. If $r=$ # of roses and $f=$ # of ~~free~~ #1 roses then $r+f=12$, and $\dfrac{r}{2}=f\Rightarrow r=8$ and $f=4$

Let $p=$ price of one rose. Then $8p+4=23.20$ or $p=\boxed{\$2.40}$

---------------- Round 2: Algebra ----------------

1. Let T = price of One T-shirt and S = price of one sweatshirt, then 5T=2S and 5T+2S=115 $\Rightarrow$ 10T = 115 or T = $\boxed{\$11.50}$.

2. $(x+1)(x+2)(x+3)-(x+1)(x+2)(x+4)+3(x+1)(x+2)$

$=(x+1)(x+2)\{(x+1)-(x+4)+(3)\}$

$=(x+1)(x+2)\{0\}=\boxed{0}$

3. $w=ksv^2 \Rightarrow k=\dfrac{w}{sv^2}$

$$w=\left(\frac{45lbs}{10ft^2(30MPH)^2}\right)(10ft^2)(80MPH)^2=\boxed{320lbs}.$$

------------------- Round 3: Set Theory -----------------

1. $A\cup B=\{2,7\}\Rightarrow$ the elements 2, 7 $\in A$ or B. $A\cap B=\{2\}\Rightarrow$ the element 2 $\in A$.

$B\cup C=\{1,2,3,4,5,6\}\Rightarrow 7\notin B$. Therefore $7\in A$ or $A=\boxed{\{2,7\}}$.

2. Looking at the differences between entries in the Number of Subsets column, $16-4=12\Rightarrow\boxed{4}$ elements.

Number of Elements	Number of Subsets
1	2
2	4
3	8
4	16

3. $22-10=\boxed{12}$

four largest $-$ four smallest counting numbers of any 7 consecutive counting numbers

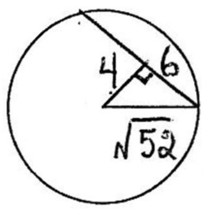
or (N+22) $-$ (N+10) = 12

------------------ Round 4: Measurement ------------------

1. Note the 4 quarter circles form a complete circle whose radius is $\frac{x}{2}$.

   Area of the star = area of the square of side x − area of circle of radius $\frac{x}{2}$ =

   $\boxed{x^2-\pi\left(\frac{x}{2}\right)^2}$ or $\boxed{x^2\left(1-\frac{\pi}{4}\right)}$.

2. 

   Use the Pythagorean theorem

   $\pi r^2=\pi\left(\sqrt{52}\right)^2=\boxed{52\pi}$

3. Use the Pythagorean theorem to get the height of the cone ($\frac{\sqrt{15}}{2}x$)

$$\frac{\textit{volume of semisphere of radius } \frac{x}{2}}{\textit{volume of cone of radius } \frac{x}{2} \textit{ \& slant height } 2x}=\frac{\frac{1}{2}\left(\frac{4}{3}\pi\left(\frac{x}{2}\right)^3\right)}{\frac{1}{3}\pi\left(\frac{x}{2}\right)^2\frac{\sqrt{15}}{2}x}=\frac{\frac{2\pi x^3}{24}}{\frac{\sqrt{15}\pi x^3}{24}}=\cancel{\boxed{\frac{2}{\sqrt{15}}}}=\boxed{\frac{2\sqrt{15}}{15}}$$

------------------- Round 5: Polynomial Equations ------------------

1. $x^2-6x+4=4 \textit{ or } x^2-6x+4=4.\ \therefore x=\boxed{0,6,2,4}$

2. sum of the roots $=-m^2$ and the product of the roots $=n^2$.
   $\therefore m+n=-m^2 \textit{ and } mn=n^2$. Since $m,n\neq 0, mn=n^2\Rightarrow m=n.$
   Substituting $m=n$ into $m+n=-m^2\Rightarrow m^2=-2m \textit{ or } m=\boxed{-2}$.

3. Let r,s,t be the roots. Looking for $\frac{1}{r}+\frac{1}{s}+\frac{1}{t}=\frac{st+rt+rs}{rst}=\boxed{\frac{3}{4}=0.75}$.

------------------- Team ---------------

1. $\frac{6-4[23]-4}{-16+1}=\frac{6-92-4}{-15}=\frac{-90}{-15}=\boxed{6}$.

2. $9a^2-24a+16-12=4a^2-5a-6+14$

   $5a^2-19a-4=0\Rightarrow(5a+1)(a-4)=0\Rightarrow a=\boxed{\frac{1}{5}=0.2} \textit{ or } a=\boxed{4}$.

---------------- Team -------------------

3. $B' = \{b,d,f\} \Rightarrow B' \cup C = \boxed{\{b,d,e,f,g\}}$.

4. Area of the square $= s^2 =$ area of the rectangle. $\Rightarrow$ width of the rectangle $\frac{S^2}{L}$.

   Therefore the width of the rectangle $= 2L + \frac{2s^2}{L}$. The circumference of the circle = the perimeter of the rectangle $\Rightarrow 2\pi R = \frac{2L+2S^2}{L}$ *or* $\pi = \boxed{\frac{L^2+S^2}{RL}}$.

5. $f(x) = a(x-3)(4x+3)(2x-5)$
   $f(2) = a(-1)(11)(-1) = 66 \Rightarrow a = 6$. Therefore $f(-4) = 6(-7)(-13)(-13) = \boxed{-7098}$.

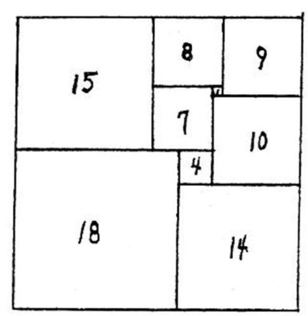
6. Note the digit 8 requires the most Roman numerals (4). So the year with the most 8's would require the most Roman numerals. Hence the year in the 21st century that would require the most Roman numerals is $\boxed{2088}$.

7. Hint use the 9th row of Pascal's triangle, double spacing, and addition. Pattern wise:

$$\begin{aligned}
101 &= 101\\
101^2 &= 10201\\
101^3 &= 1030301\\
101^4 &= 104060401\\
101^5 &= 10510100501\\
101^6 &= 1061520150601\\
101^7 &= 107213535210701\\
101^8 &= 10828567956280801\\
101^9 &= 1093685272684360901
\end{aligned}$$

Answer: $\boxed{4,360,901}$.

Possible set-up for problem 9!



8. The power of x will have to be $\left(x^n\right)^{8-p}\left(x^{-3}\right)^p$ where $p = 0,1,2,\cdots,8$. Looking for sum of the powers to be = 0. Hence $n(8-p)-3p = 0$ *or* $n = \frac{3p}{8-p}$.

p	0	1	2	3	4	5	6	7	8
$n = \frac{3p}{8-p}$	0	$\frac{3}{7}$	$\frac{6}{6} = 1$	$\frac{9}{5}$	$\frac{12}{4} = 3$	$\frac{15}{3} = 5$	$\frac{18}{2} = 9$	$\frac{21}{1} = 21$	–

So $1+3+5+9+21 = \boxed{39}$.

9. The sum of the areas of the 9 squares is 1056. Now 1056 can be factored 12 different ways using the product of two whole numbers. Since the largest square has a base of 18, the answer is restricted to $\boxed{32\ by\ 33}$ units. Note: rotation and flipping of the rectangle still gives 32 by 33 units.